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## Comparative Study of Different Approaches in the Prediction of Transverse Thermal Conductivity

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### Abstract

The difficulty associated with the transverse heat flow in the composite is that the arrangement of constituents is neither parallel nor transverse to the direction of heat flow, which limits the usage of simple Rule of Mixtures (ROM) or Inverse Rule of Mixtures (IROM) for the prediction of Transverse Thermal Conductivity ( $K_2$ ). Two different approaches have been observed from the literature to predict  $K_2$ . In one of the approaches 1-D Fourier's Law of heat conduction is applied to a control volume in the form of a unit cell neglecting the cross flow of heat within the cell. In the second method electrical analogy is applied in such a manner that it predicts the resistance in the required direction, where in the usage of 1-D principle is justified due to the elimination of cross flow within the unit cell. In the previous work of the authors, an FE model is developed and validated for the electrical analogy approach only. In the present work results obtained from the developed FE model by the authors and from the FE model developed by earlier researchers based on the first approach are compared and variation with respect to constituent proportions and conductivity is discussed.

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Keywords: Transverse thermal conductivity; FEM; Electrical analogy.

### Introduction

From the literature it is observed that there are many parameters like arrangement of fibers, volume fraction, fiber angle, ratio of fiber conductivity to matrix conductivity etc. are affecting the transverse thermal conductivity ( $K_2$ ) of

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the lamina. It is found from the literature that there are two different approaches to evaluate the transverse thermal conductivity. In the first approach, the internal anisotropy of the lamina is not considered and  $K_2$  is estimated using simple Fourier's law of 1-D heat conduction. Some of the worth mentioned studies from this criterion are Perrins *et al*(1979), who had published exact analytical and experimental results for  $K_2$  and showed very good agreement between experimental and theoretical studies. Another work using numerical studies has been made by Lu(1994), who matched their results with Perrins and stood as source of inspiration for several researches who developed FE models for  $K_2$ . Sambasiva Rao *et al*(2008) developed a 3-D finite element model for circular fibers in square unit cell and compared the results with Perrins *et al* to validate his approach.

In the second approach, Springer&Sai (1967), Behrens (1968), Mingqing Zou *et al* (2002) considered Representative Volume Element (RVE) as two segments, first one consists of fiber and matrix arranged normal to the heat flow direction and the second segment being the pure matrix above the first segment, so that the two segments remain parallel to the direction of heat flow, that facilitated them to use IROM for the first segment and ROM for the two segments. This method allows heat flow in considered direction only and the usage of 1-D Fourier's law of conduction is justified. Srinivasa Rao *et al* (2014) developed FE models in support of the second criterion.

Prior to Srinivasa Rao *et al* (2014) there was no distinction of the two approaches and the contributors of both the methods tried to convince by comparing their results irrespective of the approach. An attempt has been made in the present work to distinguish the two approaches through finite element analysis which is versatile. The results are obtained using commercial finite element software ANSYS v 14.5, and the cases where the two approaches differ considerably and the probable reasons are identified.

## 1. Finite Element Model

A schematic diagram of the unidirectional fiber composite is shown in Fig.1, where the fibers are arranged in a square array. A Representative Volume Element (RVE) in the form of a square unit cell is adopted for the present analysis. The cross-sectional area of fiber relative to the total cross-sectional area of the unit cell (Fig. 2) is a measure of the volume of fiber relative to the total volume of the composite. This fraction is an important parameter in composite materials and is called fiber volume fraction ( $V_f$ ).

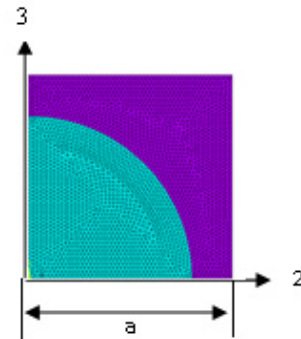
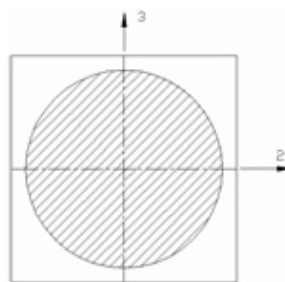
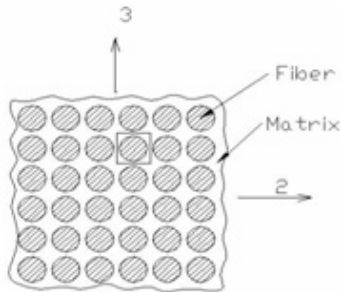


Fig. 1. Concept of unit cells

Fig. 2. Isolated unit cell of Square packed array

Fig. 3. FE model

The 1-2-3 coordinate system shown in Fig. 2 is used to study the behavior of a unit cell (The direction 1 is along the fiber axis and normal to the plane of the 2D figure shown). The isolated unit cell behaves as part of a larger array of unit cells.

It is assumed that the geometry, material and loading of the unit cell are symmetrical with respect to 1-2-3 coordinate system. Therefore, a one forth portion of the unit cell is modeled and the 2-D finite element mesh on one forth portion of the unit cell is shown in Fig. 3. The mesh is generated using six node triangular element

(PLANE-35) of ANSYS software, which is quadratic and is best suited along the curved interface between the fiber and the matrix, and has the capability of incorporating isotropic as well as orthotropic materials.

**Boundary Conditions:** Temperature boundary conditions for one-fourth model are as follows.  
sides of the unit cell is taken as '2a'.

$$T(x, 0) = T_1; T(x, a) = T_2$$

The other two faces are subjected to adiabatic boundary conditions.

The effective transverse thermal conductivity is calculated using the equation.

$$q_y = -k_2 \frac{\partial T}{\partial y}$$

Heat flux and the temperature gradient in the above equation are obtained from the finite element solution.

## Results and Discussions

The values of normalized transverse thermal conductivity ( $K_2/K_m$ ) of the composite from both the approaches (A-I, A-II) against fiber volume fraction ( $V_f$ ) at different  $K_f/K_m$  are listed in tables 1 and 2. The value of  $K_2/K_m$  is observed to be increasing with  $K_f/K_m$  and  $V_f$  as demonstrated by earlier researchers.

For all the values of  $V_f$  at smaller  $K_f/K_m$  ratios and for higher values of  $V_f$  at  $K_f/K_m$  infinity, both the approaches are found to be in reasonably good agreement. At smaller values of  $K_f/K_m$  the mismatch between the properties is low and therefore the deviation of the material from being isotropic is minimum. At higher values of  $K_f/K_m$  and  $V_f$ , the interaction between the fiber and the matrix reduces causing for heat flow close to 1-D.

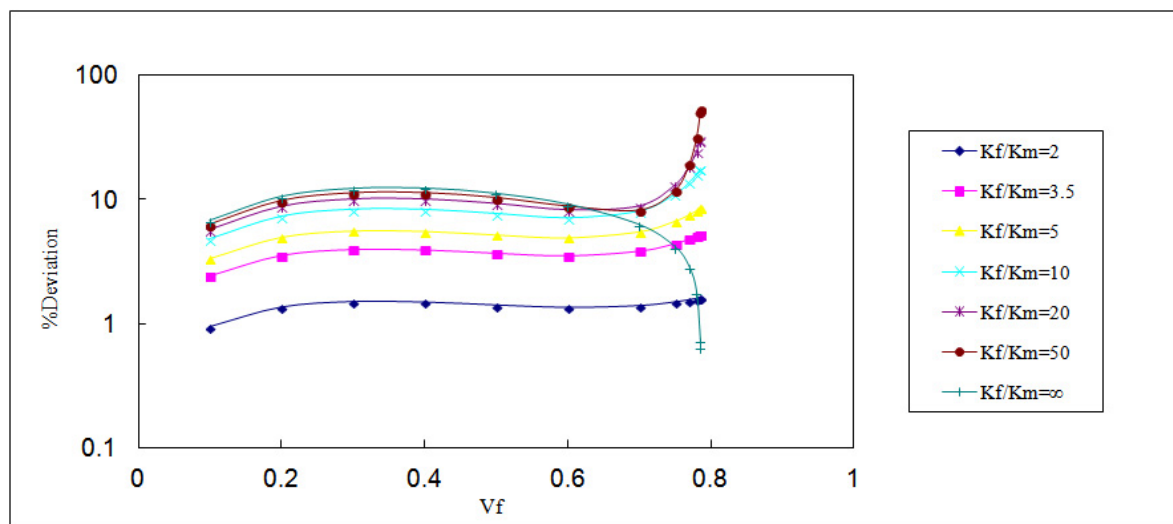
In the other cases significant deviation between the two approaches can be observed, which is due to the deviation in governing principles in the approaches. It is observed from the Fig.4 that there are fluctuations in the percentage deviation with respect to  $V_f$  at any given  $K_f/K_m$ . This might be due to the following reasons. At a given  $K_f/K_m$  increase in  $V_f$  causes for more cross flow between the fiber and matrix near the interface. At the same time due to the reduction in distance between the adjacent fibers the amount of straight flow increases. The first factor leads to more deviation in the approaches, where as the second factor reduces the deviation.

Table 1

$V_f$	$K_f/K_m=2$		$K_f/K_m=3.5$		$K_f/K_m=5$		$K_f/K_m=10$	
	A-I	A-II	A-I	A-II	A-I	A-II	A-I	A-II
0.1	1.069	1.059	1.118	1.118	1.143	1.106	1.178	1.124
0.2	1.143	1.128	1.25	1.25	1.308	1.246	1.391	1.297
0.3	1.222	1.204	1.4	1.4	1.5	1.421	1.652	1.525
0.4	1.308	1.289	1.573	1.573	1.73	1.639	1.98	1.829
0.5	1.401	1.381	1.775	1.775	2.012	1.913	2.414	2.243
0.6	1.503	1.483	2.018	2.018	2.373	2.262	3.035	2.835
0.7	1.615	1.593	2.323	2.323	2.869	2.719	4.058	3.757
0.75	1.676	1.652	2.511	2.511	3.208	3.005	4.937	4.449
0.77	1.702	1.676	2.595	2.595	3.372	3.134	5.458	4.799
0.78	1.715	1.688	2.64	2.64	3.463	3.201	5.792	4.994
0.785	1.722	1.694	2.663	2.663	3.512	3.236	5.991	5.098
0.7854	1.722	1.695	2.665	2.665	3.516	3.239	6.008	5.106

Table 2

$V_f$	$K_f/K_m=20$		$K_f/K_m=50$		$K_f/K_m=10000$ $0(\infty)$	
	A-I	A-II	A-I	A-II	A-I	A-II
0.1	1.1989	1.134	1.2125	1.1403	1.2221	1.1445
0.2	1.4419	1.326	1.4758	1.3439	1.5001	1.3566
0.3	1.7468	1.587	1.8121	1.6276	1.8597	1.6565
0.4	2.1451	1.949	2.2624	2.0312	2.3501	2.0917
0.5	2.6998	2.471	2.913	2.6387	3.0783	2.768
0.6	3.5591	3.288	3.9847	3.6598	4.3376	3.9731
0.7	5.2066	4.783	6.3249	5.8437	7.4168	6.978
0.75	6.9883	6.176	9.503	8.4954	12.688	12.18
0.77	8.3234	7.011	12.666	10.596	20.21	19.649
0.78	9.3854	7.53	16.143	12.225	34.874	34.265
0.785	10.167	7.823	20.055	13.305	102.29	101.54
0.7854	10.244	7.847	20.607	13.401	154.86	153.87

Fig. 4. Variation of % error of  $K_2$  with  $V_f$

#### 4. Conclusions

An attempt is made to distinguish the two different approaches in predicting the  $K_2$  of FRP composites. It is evident from the above results that there is considerable deviation in the results obtained from the two approaches for the range of  $V_f$  and  $K_f/K_m$  in practice.

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